

# K3 transitions tables

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## 1 Tables

We collect together all K3 transitions appearing in the web of canonical 3-folds in codimension  $\leq 3$ . The first two tables list the numerical K3 transitions and non-K3 transitions used to improve Theorem 1.3 to Theorem 1.3'. Since the contracted surface  $S$  is not quasismooth in the non-K3 case, we list the singularities occurring on  $S$ . The notation  $\frac{1}{r}(a, b, c; d)$  describes a hyperquotient singularity. That is, the quotient of a hypersurface ( $f = 0$ ) in  $\mathbb{C}^3$  by  $\mathbb{Z}/r$  acting by  $(a, b, c)$ , where  $f$  is in eigenspace  $d$ . The next three tables list canonical 3-folds grouped by codimension, and further ordered by decreasing geometric genus  $p_g$ . The raw data used to create these tables is available from

[www2.iag.uni-hannover.de/~coughlan/research/k3-transitions](http://www2.iag.uni-hannover.de/~coughlan/research/k3-transitions)

### Numerical K3 transitions

| Transition            | Contracted surface                            |
|-----------------------|---|
| $46 \rightarrow 137$  | $S_{10} \subset \mathbb{P}(1, 1, 3, 5)$       |
| $53 \rightarrow 78$   | $S_{8,12} \subset \mathbb{P}(1, 3, 4, 5, 7)$  |
| $64 \rightarrow 116$  | $S_{11,12} \subset \mathbb{P}(1, 4, 5, 6, 7)$ |
| $105 \rightarrow 136$ | $S_{8,10} \subset \mathbb{P}(1, 3, 4, 5, 5)$  |
| $106 \rightarrow 80$  | $S_{9,10} \subset \mathbb{P}(1, 3, 4, 5, 6)$  |

### Non-K3 transitions

| Transition            | Contracted surface                            | Singularities  |
|-----------------------|---|--|
| $12 \rightarrow 17$   | $S_{14} \subset \mathbb{P}(1, 3, 4, 7)$       | $\frac{1}{3}(1, 1, 1; 2), \frac{1}{4}(1, 3, 3; 2)$                             |
| $12 \rightarrow 19$   | $S_{12} \subset \mathbb{P}(1, 3, 4, 5)$       | $\frac{1}{5}(1, 3, 4; 2)$  |
| $64 \rightarrow 137$  | $S_{14} \subset \mathbb{P}(1, 3, 4, 7)$       | $\frac{1}{3}(1, 1, 1; 2), \frac{1}{4}(1, 3, 3; 2)$                             |
| $77 \rightarrow 113$  | $S_{12,18} \subset \mathbb{P}(4, 5, 6, 7, 9)$ | $\frac{1}{5}(1, 4, 4; 3), \frac{1}{7}(2, 6)$                                   |
| $109 \rightarrow 136$ | $S_{8,14} \subset \mathbb{P}(3, 4, 4, 5, 7)$  | $\frac{1}{3}(1, 1, 1; 2), 2 \times \frac{1}{4}(1, 3, 3; 2), \frac{1}{5}(2, 4)$ |

| ID | Variety                                       | Target | Via K3                                   |
|----|---|--------|--|
| 1  | $X_6 \subset \mathbb{P}^4$                    | 2      | $S_5 \subset \mathbb{P}(1, 1, 1, 2)$     |
|    |   | 6      | $S_4 \subset \mathbb{P}(1, 1, 1, 1)$     |
| 2  | $X_7 \subset \mathbb{P}^4(1, 1, 1, 1, 2)$     | 3      | $S_6 \subset \mathbb{P}(1, 1, 2, 2)$     |
|    |   | 4      | $S_6 \subset \mathbb{P}(1, 1, 1, 3)$     |
|    |   | 7      | $S_5 \subset \mathbb{P}(1, 1, 1, 2)$     |
| 6  | $X_{10} \subset \mathbb{P}^4(1, 1, 1, 1, 5)$  | —      | $1 \rightarrow 6$                        |
| 3  | $X_8 \subset \mathbb{P}^4(1, 1, 1, 2, 2)$     | 5      | $S_7 \subset \mathbb{P}(1, 1, 2, 3)$     |
|    |   | 10     | $S_6 \subset \mathbb{P}(1, 1, 2, 2)$     |
| 4  | $X_9 \subset \mathbb{P}^4(1, 1, 1, 2, 3)$     | 13     | $S_7 \subset \mathbb{P}(1, 1, 2, 3)$     |
|    |   | 9      | $S_8 \subset \mathbb{P}(1, 1, 2, 4)$     |
| 7  | $X_{12} \subset \mathbb{P}^4(1, 1, 1, 2, 6)$  | —      | $2 \rightarrow 7$                        |
|    |   | 51     | $S_{12} \subset \mathbb{P}(1, 2, 3, 6)$  |
| 5  | $X_{10} \subset \mathbb{P}^4(1, 1, 2, 2, 3)$  | 8      | $S_9 \subset \mathbb{P}(1, 2, 3, 3)$     |
|    |   | 14     | $S_8 \subset \mathbb{P}(1, 2, 2, 3)$     |
| 10 | $X_{14} \subset \mathbb{P}^4(1, 1, 2, 2, 7)$  | —      | $3 \rightarrow 10$                       |
|    |   | 56     | $S_{14} \subset \mathbb{P}(2, 2, 3, 7)$  |
| 13 | $X_{16} \subset \mathbb{P}^4(1, 1, 2, 3, 8)$  | —      | $4 \rightarrow 13$                       |
| 9  | $X_{12} \subset \mathbb{P}^4(1, 1, 2, 3, 4)$  | 12     | $S_{11} \subset \mathbb{P}(1, 2, 3, 5)$  |
|    |   | 18     | $S_{10} \subset \mathbb{P}(1, 2, 3, 4)$  |
|    |   | 19     | see §3.3                                 |
| 8  | $X_{12} \subset \mathbb{P}^4(1, 2, 2, 3, 3)$  | —      | $5 \rightarrow 8$                        |
| 14 | $X_{18} \subset \mathbb{P}^4(1, 2, 2, 3, 9)$  | —      | $5 \rightarrow 14$                       |
|    |   | 70     | $S_{18} \subset \mathbb{P}(2, 3, 4, 9)$  |
| 12 | $X_{16} \subset \mathbb{P}^4(1, 2, 3, 4, 5)$  | 16     | $S_{15} \subset \mathbb{P}(2, 3, 5, 5)$  |
|    |   | 21     | $S_{14} \subset \mathbb{P}(2, 3, 4, 5)$  |
|    |   | 15     | $S_{15} \subset \mathbb{P}(3, 3, 4, 5)$  |
|    |   | 75     | $S_{16} \subset \mathbb{P}(2, 3, 4, 7)$  |
| 18 | $X_{22} \subset \mathbb{P}^4(1, 2, 3, 4, 11)$ | —      | $9 \rightarrow 18$                       |
|    |   | 79     | $S_{22} \subset \mathbb{P}(2, 4, 5, 11)$ |
| 11 | $X_{15} \subset \mathbb{P}^4(1, 2, 3, 3, 5)$  | 15     | $S_{14} \subset \mathbb{P}(2, 3, 4, 5)$  |
|    |   | 17     | see §3.3                                 |
|    |   | 63     | $S_{15} \subset \mathbb{P}(3, 3, 4, 5)$  |
|    |   | 68     | $S_{15} \subset \mathbb{P}(2, 3, 5, 5)$  |

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| ID | Variety                                       | Target | via K3                                   |
|----|---|--------|--|
| 19 | $X_{28} \subset \mathbb{P}^4(1, 3, 4, 5, 14)$ | —      | see §3.3                                 |
|    |   | 82     | $S_{28} \subset \mathbb{P}(3, 4, 7, 14)$ |
| 17 | $X_{21} \subset \mathbb{P}^4(1, 3, 4, 5, 7)$  | 20     | $S_{20} \subset \mathbb{P}(3, 4, 5, 8)$  |
|    |   | 22     | $S_{19} \subset \mathbb{P}(3, 4, 5, 7)$  |
|    |   | 23*    | $S_{20} \subset \mathbb{P}(2, 5, 6, 7)$  |
|    |   | 80     | $S_{21} \subset \mathbb{P}(3, 5, 6, 7)$  |
| 16 | $X_{20} \subset \mathbb{P}^4(2, 3, 4, 5, 5)$  | —      | $12 \rightarrow 16$                      |
| 21 | $X_{30} \subset \mathbb{P}^4(2, 3, 4, 5, 15)$ | —      | $12 \rightarrow 21$                      |
| 15 | $X_{18} \subset \mathbb{P}^4(2, 3, 3, 4, 5)$  | —      | $11 \rightarrow 15, 12 \rightarrow 15$   |
| 20 | $X_{28} \subset \mathbb{P}^4(3, 4, 5, 7, 8)$  | —      | $17 \rightarrow 20$                      |
| 22 | $X_{40} \subset \mathbb{P}^4(3, 4, 5, 7, 20)$ | —      | $17 \rightarrow 22$                      |
| 23 | $X_{46} \subset \mathbb{P}^4(4, 5, 6, 7, 23)$ | —      | $17 \rightarrow 23$                      |

Table 1: Hypersurfaces

| ID | Variety   | Target           | Via K3   |
|----|---|------------------|--|
| 24 | $X_{3,4} \subset \mathbb{P}(1, 1, 1, 1, 1, 1)$  | 1                | $S_{2,3} \subset \mathbb{P}(1, 1, 1, 1, 1, 1)$ |
| 25 | $X_{2,5} \subset \mathbb{P}(1, 1, 1, 1, 1, 1)$  | 1                | $S_4 \subset \mathbb{P}(1, 1, 1, 1, 1, 1)$     |
| 26 | $X_{4,4} \subset \mathbb{P}(1, 1, 1, 1, 1, 2)$  | 2                | $S_{3,3} \subset \mathbb{P}(1, 1, 1, 1, 1, 2)$ |
| 27 | $X_{3,5} \subset \mathbb{P}(1, 1, 1, 1, 1, 2)$  | 2                | $S_4 \subset \mathbb{P}(1, 1, 1, 1, 1, 2)$     |
|    |   | 35               | $S_{3,3} \subset \mathbb{P}(1, 1, 1, 1, 1, 2)$ |
| 32 | $X_{2,8} \subset \mathbb{P}(1, 1, 1, 1, 1, 4)$  | 35               | $S_8 \subset \mathbb{P}(1, 1, 2, 4)$           |
| 28 | $X_{4,5} \subset \mathbb{P}(1, 1, 1, 1, 2, 2)$  | 3                | $S_{3,4} \subset \mathbb{P}(1, 1, 1, 2, 2, 2)$ |
| 29 | $X_{3,6} \subset \mathbb{P}(1, 1, 1, 1, 2, 2)$  | 3                | $S_5 \subset \mathbb{P}(1, 1, 1, 2, 2, 2)$     |
|    |   | 41               | $S_{3,4} \subset \mathbb{P}(1, 1, 1, 2, 2, 2)$ |
| 31 | $X_{4,6} \subset \mathbb{P}(1, 1, 1, 1, 2, 3)$  | 4                | $S_5 \subset \mathbb{P}(1, 1, 1, 2, 3, 3)$     |
| 35 | $X_{3,8} \subset \mathbb{P}(1, 1, 1, 1, 2, 4)$  | —                | $27 \rightarrow 35$                            |
| 30 | $X_{4,6} \subset \mathbb{P}(1, 1, 1, 2, 2, 2)$  | 38               | $S_{4,5} \subset \mathbb{P}(1, 1, 2, 2, 2, 3)$ |
| 33 | $X_{5,6} \subset \mathbb{P}(1, 1, 1, 2, 2, 3)$  | 5                | $S_{4,5} \subset \mathbb{P}(1, 1, 2, 2, 2, 3)$ |
|    |   | 42               | $S_6 \subset \mathbb{P}(1, 1, 2, 2, 2, 3)$     |
| 34 | $X_{4,7} \subset \mathbb{P}(1, 1, 1, 2, 2, 3)$  | 5                | $S_6 \subset \mathbb{P}(1, 1, 2, 2, 2, 3)$     |
|    |   | 38               | $S_{4,6} \subset \mathbb{P}(1, 2, 2, 2, 2, 3)$ |
| 37 | $X_{6,6} \subset \mathbb{P}(1, 1, 1, 2, 3, 3)$  | 39               | $S_{5,6} \subset \mathbb{P}(1, 2, 2, 3, 3, 3)$ |
|    |   | 46               | $S_{5,6} \subset \mathbb{P}(1, 1, 2, 3, 3, 4)$ |
| 41 | $X_{3,10} \subset \mathbb{P}(1, 1, 1, 2, 2, 5)$ | —                | $29 \rightarrow 41$                            |
| 44 | $X_{4,10} \subset \mathbb{P}(1, 1, 1, 2, 3, 5)$ | 48               | $S_{10} \subset \mathbb{P}(1, 1, 3, 5)$        |
| 36 | $X_{6,6} \subset \mathbb{P}(1, 1, 2, 2, 2, 3)$  | 43               | $S_{5,6} \subset \mathbb{P}(1, 2, 2, 3, 3, 3)$ |
|    |   | 92               | $S_{6,6} \subset \mathbb{P}(2, 2, 2, 3, 3, 3)$ |
| 38 | $X_{4,8} \subset \mathbb{P}(1, 1, 2, 2, 2, 3)$  | —                | $30 \rightarrow 38, 34 \rightarrow 38$         |
| 39 | $X_{6,7} \subset \mathbb{P}(1, 1, 2, 2, 3, 3)$  | 8                | $S_{5,6} \subset \mathbb{P}(1, 2, 2, 3, 3, 3)$ |
|    |   | 47               | $S_{6,6} \subset \mathbb{P}(1, 2, 3, 3, 3, 3)$ |
| 40 | $X_{4,9} \subset \mathbb{P}(1, 1, 2, 2, 3, 3)$  | 8                | $S_8 \subset \mathbb{P}(1, 2, 2, 3, 3, 3)$     |
| 42 | $X_{6,8} \subset \mathbb{P}(1, 1, 2, 2, 3, 4)$  | —                | $33 \rightarrow 42$                            |
| 45 | $X_{4,10} \subset \mathbb{P}(1, 1, 2, 2, 2, 5)$ | 50               | $S_{10} \subset \mathbb{P}(1, 2, 2, 2, 5)$     |
|    |   | 56               | $S_8 \subset \mathbb{P}(1, 2, 2, 2, 5)$        |
| 46 | $X_{6,9} \subset \mathbb{P}(1, 1, 2, 3, 3, 4)$  | —                | $37 \rightarrow 46$                            |
|    |   | 67               | $S_{6,7} \subset \mathbb{P}(1, 2, 3, 3, 3, 4)$ |
|    |   | 137 <sup>†</sup> | $S_{10} \subset \mathbb{P}(1, 1, 3, 5)$        |

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| ID | Variety  | Target | via K3  |
|----|--|--------|---|
| 48 | $X_{6,10} \subset \mathbb{P}(1, 1, 2, 3, 3, 5)$  | 11     | $S_9 \subset \mathbb{P}(1, 2, 3, 3)$          |
|    |  | 71     | $S_{6,8} \subset \mathbb{P}(1, 2, 3, 3, 5)$   |
| 51 | $X_{4,12} \subset \mathbb{P}(1, 1, 2, 2, 3, 6)$  | —      | $7 \rightarrow 51$                            |
| 43 | $X_{6,8} \subset \mathbb{P}(1, 2, 2, 2, 3, 3)$   | 61     | $S_{6,6} \subset \mathbb{P}(2, 2, 2, 3, 3)$   |
|    |  | 101    | $S_{6,8} \subset \mathbb{P}(2, 2, 3, 3, 4)$   |
| 47 | $X_{6,9} \subset \mathbb{P}(1, 2, 2, 3, 3, 3)$   | —      | $39 \rightarrow 47$                           |
|    |  | 55     | $S_{6,8} \subset \mathbb{P}(2, 2, 3, 3, 4)$   |
| 49 | $X_{6,10} \subset \mathbb{P}(1, 2, 2, 3, 3, 4)$  | 70     | $S_{6,8} \subset \mathbb{P}(2, 2, 3, 3, 4)$   |
| 50 | $X_{6,10} \subset \mathbb{P}(1, 2, 2, 2, 3, 5)$  | —      | $45 \rightarrow 50$                           |
|    |  | 104    | $S_{6,10} \subset \mathbb{P}(2, 2, 3, 4, 5)$  |
| 52 | $X_{8,10} \subset \mathbb{P}(1, 2, 2, 3, 4, 5)$  | 59     | $S_{8,9} \subset \mathbb{P}(2, 3, 3, 4, 5)$   |
|    |  | 69     | $S_{6,10} \subset \mathbb{P}(2, 2, 3, 4, 5)$  |
|    |  | 107    | $S_{8,10} \subset \mathbb{P}(2, 3, 4, 4, 5)$  |
| 53 | $X_{6,12} \subset \mathbb{P}(1, 2, 3, 3, 4, 4)$  | 65     | $S_{12} \subset \mathbb{P}(2, 3, 3, 4)$       |
|    |  | 72*    | $S_{12} \subset \mathbb{P}(2, 2, 3, 5)$       |
|    |  | 78†    | $S_{8,12} \subset \mathbb{P}(1, 3, 4, 5, 7)$  |
| 54 | $X_{6,12} \subset \mathbb{P}(1, 2, 2, 3, 4, 5)$  | 59     | $S_{12} \subset \mathbb{P}(2, 3, 3, 4)$       |
|    |  | 66     | $S_{12} \subset \mathbb{P}(2, 2, 3, 5)$       |
| 56 | $X_{4,14} \subset \mathbb{P}(1, 2, 2, 2, 3, 7)$  | —      | $10 \rightarrow 56, 45 \rightarrow 56$        |
|    |  | 61     | $S_{14} \subset \mathbb{P}(2, 2, 3, 7)$       |
| 57 | $X_{9,10} \subset \mathbb{P}(1, 2, 3, 3, 4, 5)$  | 15     | $S_{8,9} \subset \mathbb{P}(2, 3, 3, 4, 5)$   |
|    |  | 65     | $S_{8,10} \subset \mathbb{P}(2, 3, 4, 4, 5)$  |
|    |  | 108    | $S_{9,10} \subset \mathbb{P}(2, 3, 4, 5, 5)$  |
| 58 | $X_{6,13} \subset \mathbb{P}(1, 2, 3, 3, 4, 5)$  | 15     | $S_{12} \subset \mathbb{P}(2, 3, 3, 4)$       |
| 60 | $X_{8,12} \subset \mathbb{P}(1, 2, 3, 4, 4, 5)$  | 79     | $S_{8,10} \subset \mathbb{P}(2, 3, 4, 4, 5)$  |
| 62 | $X_{6,14} \subset \mathbb{P}(1, 2, 2, 3, 4, 7)$  | 69     | $S_{14} \subset \mathbb{P}(2, 2, 3, 7)$       |
|    |  | 70     | $S_{12} \subset \mathbb{P}(2, 3, 3, 4)$       |
| 64 | $X_{10,12} \subset \mathbb{P}(1, 2, 3, 4, 5, 6)$ | 75     | $S_{9,12} \subset \mathbb{P}(2, 3, 4, 5, 7)$  |
|    |  | 76     | $S_{10,11} \subset \mathbb{P}(2, 3, 4, 5, 7)$ |
|    |  | 77     | $S_{8,12} \subset \mathbb{P}(2, 3, 4, 5, 6)$  |
|    |  | 109    | $S_{10,12} \subset \mathbb{P}(2, 4, 5, 5, 6)$ |
|    |  | 110    | $S_{10,12} \subset \mathbb{P}(3, 3, 4, 5, 7)$ |

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| ID | Variety   | Target           | via K3  |
|----|---|------------------|---|
|    |   | 111              | $S_{10,12} \subset \mathbb{P}(2, 3, 5, 5, 7)$             |
|    |   | 112              | $S_{10,12} \subset \mathbb{P}(2, 3, 4, 5, 8)$             |
|    |   | 116 <sup>†</sup> | $S_{11,12} \subset \mathbb{P}(1, 4, 5, 6, 7)$             |
| 67 | $X_{6,16} \subset \mathbb{P}(1, 2, 3, 3, 4, 8)$   | —                | $46 \rightarrow 67$                                       |
| 71 | $X_{6,18} \subset \mathbb{P}(1, 2, 3, 3, 5, 9)$   | —                | $48 \rightarrow 71$                                       |
| 73 | $X_{6,20} \subset \mathbb{P}(1, 2, 3, 4, 5, 10)$  | 21*              | $S_{18} \subset \mathbb{P}(2, 3, 4, 9)$                   |
| 74 | $X_{12,15} \subset \mathbb{P}(1, 3, 4, 5, 6, 7)$  | 78               | $S_{12,14} \subset \mathbb{P}(4, 4, 5, 6, 7)$             |
|    |   | 81               | $S_{12,14} \subset \mathbb{P}(3, 4, 5, 7, 7)$             |
|    |   | 82               | $S_{12,13} \subset \mathbb{P}(3, 4, 5, 6, 7)$             |
|    |   | 114              | $S_{12,15} \subset \mathbb{P}(3, 4, 5, 7, 8)$             |
|    |   | 115              | $S_{12,15} \subset \mathbb{P}(3, 4, 5, 6, 9)$             |
| 55 | $X_{6,12} \subset \mathbb{P}(2, 2, 3, 3, 3, 4)$   | —                | $47 \rightarrow 55$                                       |
| 59 | $X_{8,12} \subset \mathbb{P}(2, 2, 3, 3, 4, 5)$   | —                | $52 \rightarrow 59, 54 \rightarrow 59$                    |
| 61 | $X_{6,14} \subset \mathbb{P}(2, 2, 2, 3, 3, 7)$   | —                | $43 \rightarrow 61, 56 \rightarrow 61$                    |
| 63 | $X_{6,15} \subset \mathbb{P}(2, 3, 3, 3, 4, 5)$   | —                | $11 \rightarrow 63$                                       |
| 65 | $X_{10,12} \subset \mathbb{P}(2, 3, 3, 4, 4, 5)$  | —                | $53 \rightarrow 65, 57 \rightarrow 65$                    |
| 66 | $X_{10,12} \subset \mathbb{P}(2, 2, 3, 4, 5, 5)$  | —                | $54 \rightarrow 66$                                       |
| 68 | $X_{8,15} \subset \mathbb{P}(2, 3, 3, 4, 5, 5)$   | —                | $11 \rightarrow 68$                                       |
| 69 | $X_{10,14} \subset \mathbb{P}(2, 2, 3, 4, 5, 7)$  | —                | $62 \rightarrow 69, 52 \rightarrow 69$                    |
| 70 | $X_{6,18} \subset \mathbb{P}(2, 2, 3, 3, 4, 9)$   | —                | $14 \rightarrow 70, 49 \rightarrow 70, 62 \rightarrow 70$ |
| 72 | $X_{12,14} \subset \mathbb{P}(2, 3, 4, 4, 5, 7)$  | —                | $53 \rightarrow 72$                                       |
| 75 | $X_{12,16} \subset \mathbb{P}(2, 3, 4, 5, 6, 7)$  | —                | $12 \rightarrow 75, 64 \rightarrow 75$                    |
| 76 | $X_{10,18} \subset \mathbb{P}(2, 3, 4, 5, 6, 7)$  | —                | $64 \rightarrow 76$                                       |
| 77 | $X_{12,18} \subset \mathbb{P}(2, 3, 4, 5, 6, 9)$  | —                | $64 \rightarrow 77$                                       |
| 78 | $X_{12,18} \subset \mathbb{P}(3, 4, 4, 5, 6, 7)$  | —                | $74 \rightarrow 78, 53 \rightarrow 78$                    |
| 79 | $X_{8,22} \subset \mathbb{P}(2, 3, 4, 4, 5, 11)$  | —                | $18 \rightarrow 79$                                       |
| 80 | $X_{10,21} \subset \mathbb{P}(3, 4, 5, 5, 6, 7)$  | —                | $17 \rightarrow 80$                                       |
| 81 | $X_{12,21} \subset \mathbb{P}(3, 4, 5, 6, 7, 7)$  | —                | $74 \rightarrow 81$                                       |
| 82 | $X_{12,28} \subset \mathbb{P}(3, 4, 5, 6, 7, 14)$ | —                | $19 \rightarrow 82, 74 \rightarrow 82$                    |

Table 2: Codimension 2

| ID  | Variety   | Target | via K3  |
|-----|---|--------|---|
| 117 | $X_{2,3,3} \subset \mathbb{P}(1, 1, 1, 1, 1, 1, 1)$     | 24     | $S_{2,3} \subset \mathbb{P}(1, 1, 1, 1, 1)$             |
| 118 | $X_{2,2,4} \subset \mathbb{P}(1, 1, 1, 1, 1, 1, 1)$     | 24     | $S_4 \subset \mathbb{P}(1, 1, 1, 1)$                    |
|     |   | 84     | $S_{2,2,2} \subset \mathbb{P}(1, 1, 1, 1, 1, 1)$        |
| 120 | $X_{3,3,3,3,4} \subset \mathbb{P}(1, 1, 1, 1, 1, 1, 1)$ | 24     | $S_{2,2,2} \subset \mathbb{P}(1, 1, 1, 1, 1, 1)$        |
| 84  | $X_{2,2,6} \subset \mathbb{P}(1, 1, 1, 1, 1, 1, 3)$     | —      | $118 \rightarrow 84$                                    |
| 119 | $X_{3,3,3} \subset \mathbb{P}(1, 1, 1, 1, 1, 1, 2)$     | 27     | $S_{2,3} \subset \mathbb{P}(1, 1, 1, 1, 1)$             |
| 121 | $X_{3,3,4,4,4} \subset \mathbb{P}(1, 1, 1, 1, 1, 1, 2)$ | 26     | $S_{2,3} \subset \mathbb{P}(1, 1, 1, 1, 1)$             |
| 83  | $X_{3,3,4} \subset \mathbb{P}(1, 1, 1, 1, 1, 2, 2)$     | 28     | $S_4 \subset \mathbb{P}(1, 1, 1, 1)$                    |
| 122 | $X_{3,4,4,4,5} \subset \mathbb{P}(1, 1, 1, 1, 1, 2, 2)$ | 28     | $S_{3,3} \subset \mathbb{P}(1, 1, 1, 1, 2)$             |
| 123 | $X_{4,4,4,4,4} \subset \mathbb{P}(1, 1, 1, 1, 1, 2, 2)$ | 28     | $S_{3,3,3,3,4} \subset \mathbb{P}(1, 1, 1, 1, 2, 2)$    |
| 85  | $X_{3,4,4} \subset \mathbb{P}(1, 1, 1, 1, 2, 2, 2)$     | 30     | $S_{3,4} \subset \mathbb{P}(1, 1, 1, 2, 2, 2)$          |
|     |   | 87     | $S_{4,4} \subset \mathbb{P}(1, 1, 2, 2, 2, 2)$          |
| 86  | $X_{4,4,4} \subset \mathbb{P}(1, 1, 1, 1, 2, 2, 3)$     | 34     | $S_{3,4} \subset \mathbb{P}(1, 1, 1, 2, 2, 2)$          |
| 124 | $X_{4,4,4,5,5} \subset \mathbb{P}(1, 1, 1, 1, 2, 2, 2)$ | 30     | $S_{3,3,4,4,4} \subset \mathbb{P}(1, 1, 1, 2, 2, 2, 2)$ |
| 125 | $X_{4,4,5,5,6} \subset \mathbb{P}(1, 1, 1, 1, 2, 2, 3)$ | 33     | $S_{3,4} \subset \mathbb{P}(1, 1, 1, 2, 2, 2)$          |
| 87  | $X_{4,4,4} \subset \mathbb{P}(1, 1, 1, 2, 2, 2, 2)$     | —      | $85 \rightarrow 87$                                     |
|     |   | 90     | $S_{4,4} \subset \mathbb{P}(1, 1, 2, 2, 2, 2)$          |
| 88  | $X_{4,4,5} \subset \mathbb{P}(1, 1, 1, 2, 2, 2, 3)$     | 38     | $S_{4,4} \subset \mathbb{P}(1, 1, 2, 2, 2, 2)$          |
| 89  | $X_{4,4,6} \subset \mathbb{P}(1, 1, 1, 2, 2, 3, 3)$     | 39     | $S_6 \subset \mathbb{P}(1, 1, 2, 2, 2)$                 |
| 126 | $X_{4,5,5,6,6} \subset \mathbb{P}(1, 1, 1, 2, 2, 2, 3)$ | 36     | $S_{4,4} \subset \mathbb{P}(1, 1, 2, 2, 2, 2)$          |
| 127 | $X_{4,5,6,6,7} \subset \mathbb{P}(1, 1, 1, 2, 2, 3, 3)$ | 39     | $S_{4,5} \subset \mathbb{P}(1, 1, 2, 2, 3)$             |
| 128 | $X_{5,5,6,6,6} \subset \mathbb{P}(1, 1, 1, 2, 2, 3, 3)$ | 39     | $S_{4,4,5,5,6} \subset \mathbb{P}(1, 1, 2, 2, 3, 3)$    |
| 90  | $X_{4,4,6} \subset \mathbb{P}(1, 1, 2, 2, 2, 2, 3)$     | —      | $87 \rightarrow 90$                                     |
| 91  | $X_{4,5,6} \subset \mathbb{P}(1, 1, 2, 2, 2, 3, 3)$     | 43     | $S_{4,6} \subset \mathbb{P}(1, 2, 2, 2, 3)$             |
|     |   | 95     | $S_{5,6} \subset \mathbb{P}(1, 2, 2, 3, 3)$             |
| 93  | $X_{4,6,6} \subset \mathbb{P}(1, 1, 2, 2, 3, 3, 3)$     | 47     | $S_{5,6} \subset \mathbb{P}(1, 2, 2, 3, 3)$             |
|     |   | 98     | $S_{6,6} \subset \mathbb{P}(1, 2, 3, 3, 3)$             |
|     |   | 99     | $S_{6,6} \subset \mathbb{P}(1, 2, 2, 3, 4)$             |
| 94  | $X_{5,6,6} \subset \mathbb{P}(1, 1, 2, 2, 3, 3, 4)$     | 49     | $S_{5,6} \subset \mathbb{P}(1, 2, 2, 3, 3)$             |
| 102 | $X_{6,6,8} \subset \mathbb{P}(1, 1, 2, 3, 3, 4, 5)$     | 58     | $S_{6,7} \subset \mathbb{P}(1, 2, 3, 3, 4)$             |
| 129 | $X_{5,6,6,6,7} \subset \mathbb{P}(1, 1, 2, 2, 2, 3, 3)$ | 43     | $S_{4,5,5,6,6} \subset \mathbb{P}(1, 2, 2, 2, 3, 3)$    |
| 130 | $X_{6,6,6,7,7} \subset \mathbb{P}(1, 1, 2, 2, 3, 3, 3)$ | 47     | $S_{5,5,6,6,6} \subset \mathbb{P}(1, 2, 2, 3, 3, 3)$    |
| 131 | $X_{6,6,7,7,8} \subset \mathbb{P}(1, 1, 2, 2, 3, 3, 4)$ | 49     | $S_{5,6,6,6,7} \subset \mathbb{P}(1, 2, 2, 3, 3, 4)$    |
| 132 | $X_{6,7,8} \subset \mathbb{P}(1, 1, 2, 3, 3, 4, 5)$     | 57     | $S_{6,7} \subset \mathbb{P}(1, 2, 3, 3, 4)$             |

...continued from previous page

| ID  | Variety  | Target           | via K3  |
|-----|--|------------------|---|
| 92  | $X_{4,6,6} \subset \mathbb{P}(1, 2, 2, 2, 2, 3, 3)$          | —                | 36 $\rightarrow$ 92                                   |
|     |  | 97               | $S_{6,6} \subset \mathbb{P}(2, 2, 2, 3, 3)$           |
| 95  | $X_{5,6,6} \subset \mathbb{P}(1, 2, 2, 2, 3, 3, 3)$          | —                | 91 $\rightarrow$ 95                                   |
| 96  | $X_{6,6,6} \subset \mathbb{P}(1, 2, 2, 2, 3, 3, 4)$          | 104              | $S_{6,6} \subset \mathbb{P}(2, 2, 2, 3, 3)$           |
| 98  | $X_{6,6,6} \subset \mathbb{P}(1, 2, 2, 3, 3, 3, 3)$          | —                | 93 $\rightarrow$ 98                                   |
| 99  | $X_{6,6,7} \subset \mathbb{P}(1, 2, 2, 3, 3, 3, 4)$          | —                | 93 $\rightarrow$ 99                                   |
| 100 | $X_{6,6,8} \subset \mathbb{P}(1, 2, 2, 3, 3, 4, 4)$          | 107              | $S_{6,8} \subset \mathbb{P}(2, 2, 3, 3, 4)$           |
| 103 | $X_{6,7,8} \subset \mathbb{P}(1, 2, 2, 3, 3, 4, 5)$          | 59               | $S_{6,8} \subset \mathbb{P}(2, 2, 3, 3, 4)$           |
| 105 | $X_{6,8,9} \subset \mathbb{P}(1, 2, 3, 3, 4, 4, 5)$          | 108              | $S_{8,9} \subset \mathbb{P}(2, 3, 3, 4, 5)$           |
|     |  | 136 <sup>†</sup> | $S_{8,10} \subset \mathbb{P}(1, 3, 4, 5, 5)$          |
| 106 | $X_{6,8,10} \subset \mathbb{P}(1, 2, 3, 3, 4, 5, 5)$         | 68               | $S_{8,9} \subset \mathbb{P}(2, 3, 3, 4, 5)$           |
|     |  | 80 <sup>†</sup>  | $S_{9,10} \subset \mathbb{P}(1, 3, 4, 5, 6)$          |
| 133 | $X_{7,8,8,9,10} \subset \mathbb{P}(1, 2, 2, 3, 3, 4, 5)$     | 59               | $S_{6,7,8,8,9} \subset \mathbb{P}(2, 2, 3, 3, 4, 5)$  |
| 134 | $X_{8,9,9,10,10} \subset \mathbb{P}(1, 2, 3, 3, 4, 4, 5)$    | 65               | $S_{7,8,8,9,10} \subset \mathbb{P}(2, 3, 3, 4, 4, 5)$ |
| 135 | $X_{8,9,10,10,11} \subset \mathbb{P}(1, 2, 3, 3, 4, 5, 5)$   | 68               | $S_{8,8,9,9,10} \subset \mathbb{P}(2, 3, 3, 4, 5, 5)$ |
| 137 | $X_{12,13,14,15,16} \subset \mathbb{P}(1, 3, 4, 5, 6, 7, 8)$ | —                | 46 $\rightarrow$ 137                                  |
| 97  | $X_{6,6,6} \subset \mathbb{P}(2, 2, 2, 2, 3, 3, 3)$          | —                | 92 $\rightarrow$ 97                                   |
| 101 | $X_{6,6,8} \subset \mathbb{P}(2, 2, 2, 3, 3, 3, 4)$          | —                | 43 $\rightarrow$ 101                                  |
| 104 | $X_{6,6,10} \subset \mathbb{P}(2, 2, 2, 3, 3, 4, 5)$         | —                | 50 $\rightarrow$ 104                                  |
| 107 | $X_{6,8,10} \subset \mathbb{P}(2, 2, 3, 3, 4, 4, 5)$         | —                | 52 $\rightarrow$ 107                                  |
| 108 | $X_{8,9,10} \subset \mathbb{P}(2, 3, 3, 4, 4, 5, 5)$         | —                | 57 $\rightarrow$ 108                                  |
| 109 | $X_{8,10,12} \subset \mathbb{P}(2, 3, 4, 4, 5, 5, 6)$        | —                | 64 $\rightarrow$ 109                                  |
| 110 | $X_{9,10,12} \subset \mathbb{P}(2, 3, 3, 4, 5, 6, 7)$        | —                | 64 $\rightarrow$ 110                                  |
| 111 | $X_{10,11,12} \subset \mathbb{P}(2, 3, 4, 5, 5, 6, 7)$       | —                | 64 $\rightarrow$ 111                                  |
| 112 | $X_{10,12,14} \subset \mathbb{P}(2, 3, 4, 5, 6, 7, 8)$       | —                | 64 $\rightarrow$ 112                                  |
| 113 | $X_{10,12,18} \subset \mathbb{P}(3, 4, 5, 5, 6, 7, 9)$       | —                | No numerical links                                    |
| 114 | $X_{12,14,15} \subset \mathbb{P}(3, 4, 5, 6, 7, 7, 8)$       | —                | 74 $\rightarrow$ 114                                  |
| 115 | $X_{12,15,16} \subset \mathbb{P}(3, 4, 5, 6, 7, 8, 9)$       | —                | 74 $\rightarrow$ 115                                  |
| 116 | $X_{12,16,18} \subset \mathbb{P}(4, 5, 6, 6, 7, 8, 9)$       | —                | 64 $\rightarrow$ 116                                  |
| 136 | $X_{12,13,14,15,16} \subset \mathbb{P}(3, 4, 4, 5, 5, 6, 7)$ | —                | 105 $\rightarrow$ 136                                 |

Table 3: Codimension 3